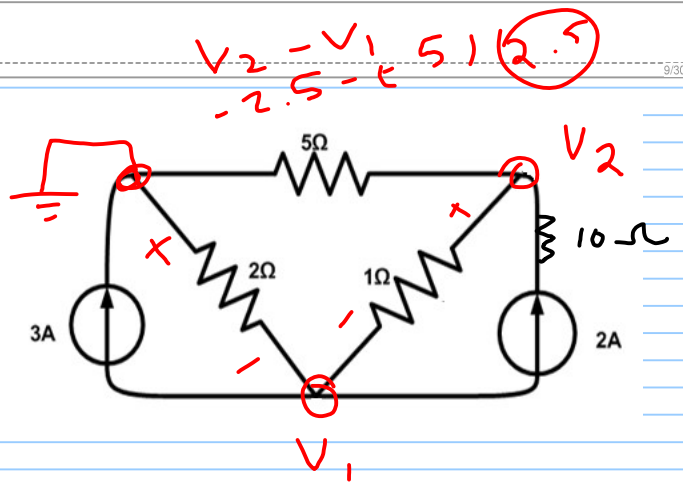


$$\underline{\underline{CN = 4}}$$

Nodal
method

@ node V_1

All currents are out }
OR " ~ " IN }



$$\underline{I_{out}} \quad 3 + \frac{V_1}{2} + \frac{V_1 - V_2}{1} + 2 = 0$$

$$\sum I_{out} = 0 \quad \text{OR}$$

$$\sum I_{IN} = 0$$

$$(1.5) V_1 - V_2 = -5 \quad \text{--- (1)}$$

@ Node V_2

$$\underline{I_{out}} \quad -2 + \frac{V_2 - V_1}{1} + \frac{V_2}{5} = 0$$

$$-V_1 + 1.2 V_2 = 2 \quad \text{--- (2)}$$

$$V_1 = -5 \text{ V}$$

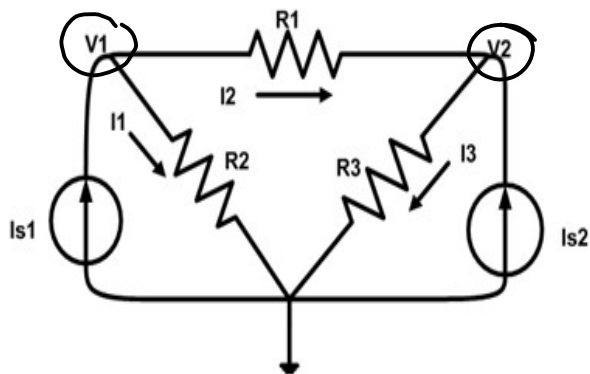
$$V_2 = -2.5 \text{ V}$$

@ node V_1

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 - \left(\frac{1}{R_1}\right) V_2 = I_{S1}$$

@ node V_2

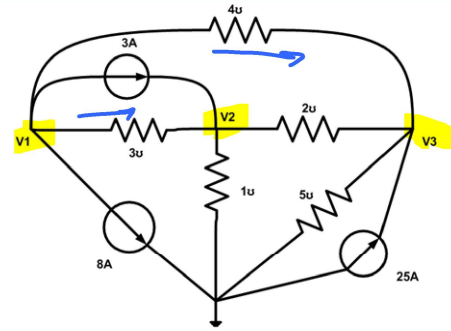
$$-\left(\frac{1}{R_1}\right) V_1 + \left(\frac{1}{R_1} + \frac{1}{R_3}\right) V_2 = I_{S2}$$



@ node 1

$$\sum I_{out} = \sum I_{in}$$

$$7V_1 - 3V_2 - 4V_3 = -11 \quad \text{--- (1)}$$



@ node V_2

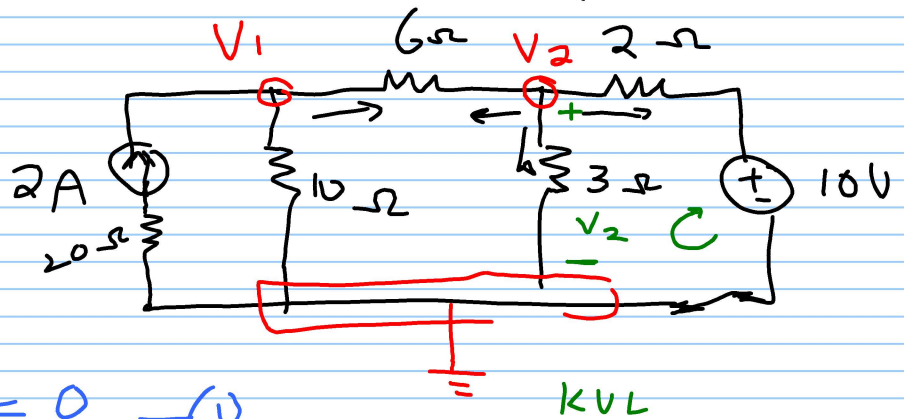
$$3(V_1 - V_2) + 4(V_1 - V_3) + 3 + 8 = 0$$

$$-(3)V_1 + (3+2+1)V_2 - (2)V_3 = 3 \quad \text{--- (2)}$$

@ node V_3

$$-4V_1 - 2V_2 + (5+2+4)V_3 = 25 \quad \text{--- (3)}$$

EX



$$\sum I_{out} = \sum I_{in}$$

node 1

$$-2 + \frac{V_1}{10} + \frac{V_1 - V_2}{6} = 0 \quad \text{--- (1)}$$

node 2

$$\sum I_{out} = 0$$

$$\frac{V_2 - V_1}{6} + \frac{V_2}{3} + \frac{V_2 - 10}{2} = 0 \quad \text{--- (2)}$$

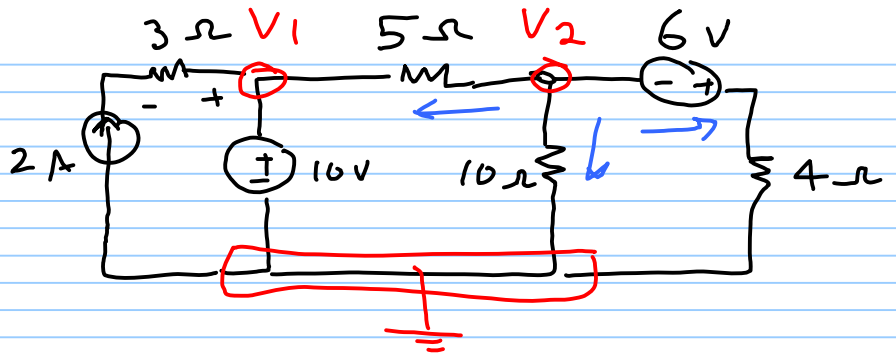
$$-V_2 + 2I_x + 10 = 0$$

$$I_x = \frac{V_2 - 10}{2}$$

$$V_1 = \text{---}$$

$$V_2 = \text{---} \quad \checkmark$$

EX1



$$(-2) + (V_1 - 10) \quad \times$$

$$V_1 = 10 \text{ V} \quad (\text{yep})$$

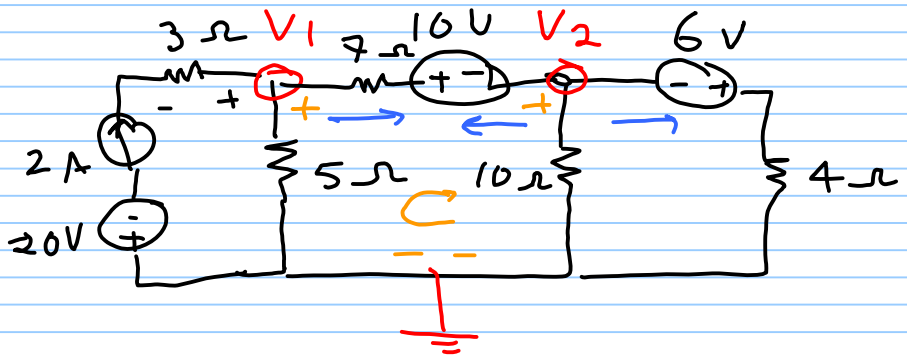
@ node V2

$$\frac{V_2 - 10}{5} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0$$

$$0.55 V_2 = 2 - 1.5$$

$$V_2 = \frac{0.5}{0.55} = 0.909 \text{ Volt}$$

EX1



$$\sum I_{out} = 2 \text{ amp}$$

@ node V1

$$-2 + \frac{V_1}{5} + \frac{V_1 - 10 - V_2}{7} = 0 \quad \text{--- (1)}$$

@ node V2

$$\frac{V_2 - V_1 + 10}{7} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0 \quad \text{--- (2)}$$

Special case

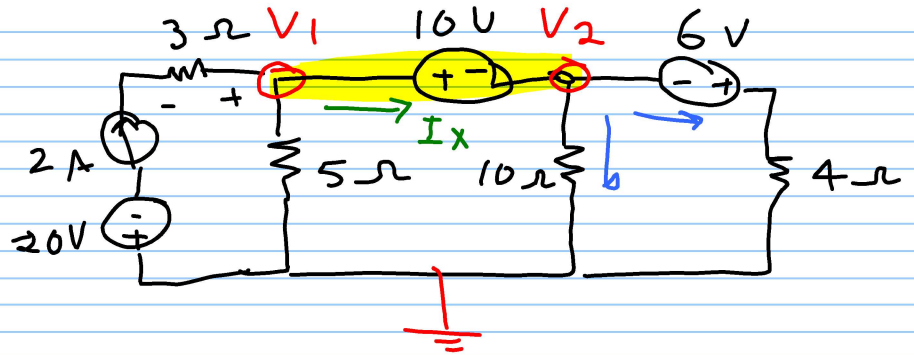
EX

@ node V_1

$$-2 + \frac{V_1}{5} + \underline{I_x} = 0$$

$$-2 + \frac{V_1}{5} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0 \quad \text{--- (1) KVL}$$

$$\underline{V_1 - V_2 = 10} \quad \text{--- (2) KVL @ supernode}$$



EX

$$I_o = \frac{V_2}{3k} \quad \text{--- (1)}$$

@ node V_1

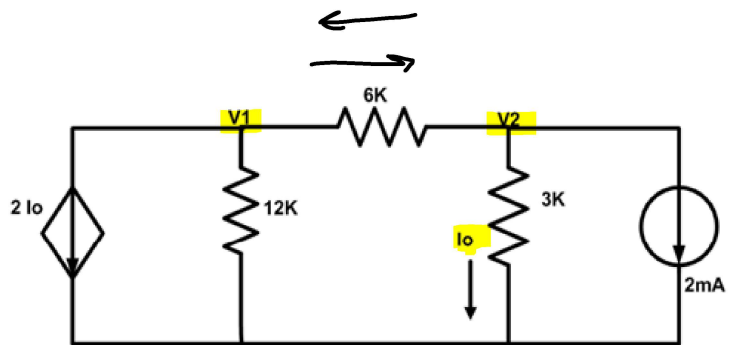
$$2I_o + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0$$

$$2\left(\frac{V_2}{3k}\right) + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0 \quad \text{--- (1) } \checkmark$$

@ node V_2

$$\frac{V_2 - V_1}{6k} + \frac{V_2}{3k} + 2mA = 0 \quad \text{--- (2) } \checkmark$$

$$V_1 = -\frac{24}{5} V, \quad V_2 = \frac{12}{5} V$$



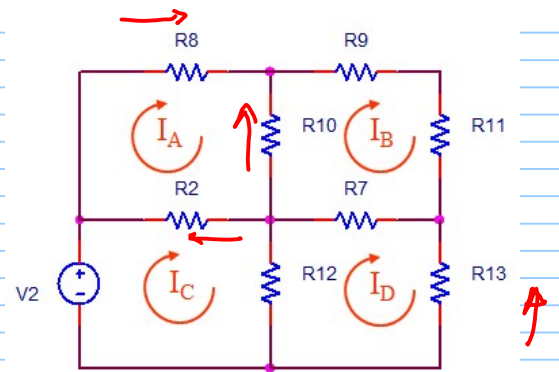
Mesh method

$$I_{R_3} = I_A$$

$$I_{R_2} = I_A - I_C$$

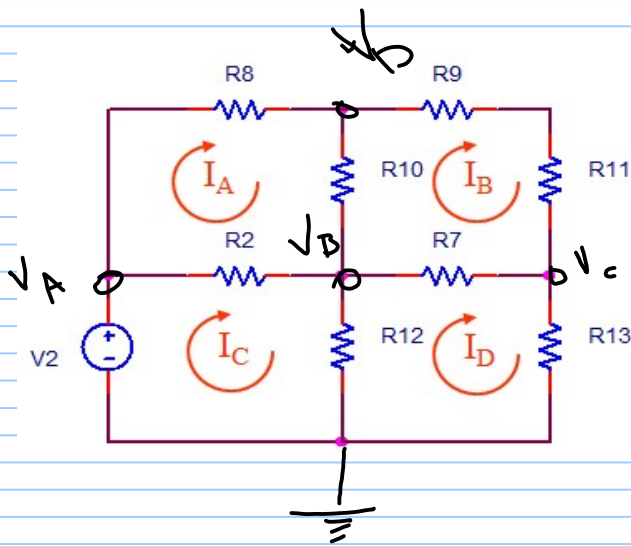
$$I_{R_{13}} = -I_D$$

$$I_{R_{10}} = I_B - I_A$$



4 equations

4 unknowns I_A, I_B, I_C
 ~~I_D~~



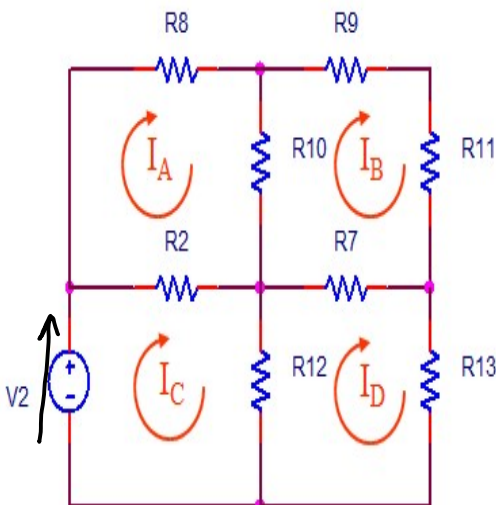
$$V_A = V_2$$

Nodal method

3 eqs

3 unknowns

$$V_B, V_C, V_D$$



Mesh KVL

$$R_8 I_A + R_{10} (I_A - I_B) + R_2 (I_A - I_C) = 0$$

$$(R_8 + R_{10} + R_2) I_A - R_{10} I_B - R_2 I_C = 0 \quad (1)$$

$$- (R_{10}) I_A + (R_{10} + R_9 + R_{11} + R_7) I_B$$

$$- 0 I_C - R_7 I_D = 0 \quad (2)$$

$$-V_2 + R_2 (I_C - I_A) + R_{12} (I_C - I_D) = 0$$

$$(R_2 + R_{12}) I_C - R_2 I_A - R_{12} I_D = V_2 \quad (3)$$

$$(R_{12} + R_7 + R_{13})I_D - 0I_A - R_7I_B - R_{12}I_C = 0 \quad \text{--- (4)}$$

EX]

$$-42 + 6I_1 + 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 42 \quad \text{--- (1)}$$

$$-3I_1 + 7I_2 = 10 \quad \text{--- (2)}$$

6A, 4A

special case 1 | current source exists
only in one mesh

EX]

$$I_2 = -5A \quad \checkmark$$

$$-10 + 4I_1 + 6(I_1 - 5) = 0$$

$$I_1 = -2A \quad \checkmark$$

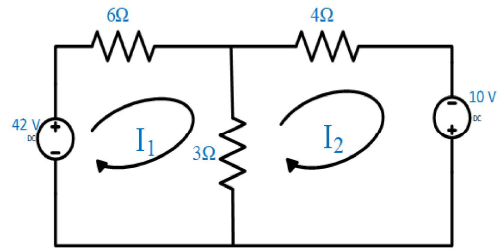


Figure 3: example 1 of mesh analysis

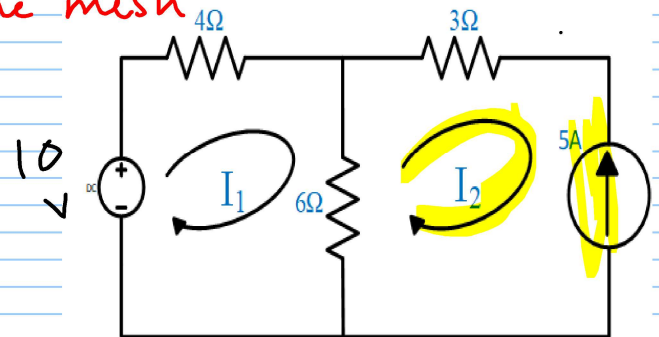


Figure 6: mesh with current source.

Case 2:

Current source exists between two meshes, a **Super mesh** is obtained

→ KVL for mesh (2)

$$(1+2+3)I_2 - (1)I_1 - (3)I_3 = 0 \text{ --- (1)}$$

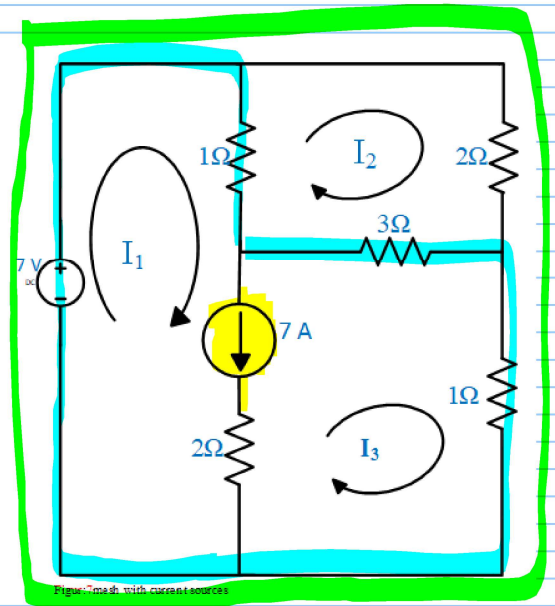
→ constraint equation

$$I_1 - I_3 = 7 \text{ --- (2)}$$

→ **super mesh equation**

$$-7 + (1)(I_1 - I_2) + (3)(I_3 - I_2) + (1)I_3 = 0$$

$$I_1 - 4I_2 + 4I_3 = 7 \text{ --- (3)}$$



$$I_3 = -7A \times$$

OR

$$-7 + 2I_2 + I_3 = 0 \text{ --- (3)*}$$

EX Mesh Analysis with dependent sources

$$V_x = (3)(I_3 - I_2)$$

$$I_1 = 15A \checkmark$$

→ constraint equation

$$\frac{V_x}{9} = I_3 - I_1$$

$$\frac{3}{9}(I_3 - I_2) = I_3 - 15$$

$$\frac{2}{9}I_2 + \frac{2}{3}I_3 = 15$$

$$I_2 + 2I_3 = 45 \text{ --- (1)}$$

$$(1)(I_2 - 15) + (2)I_2 + (3)(I_2 - I_3) = 0$$

$$6I_2 - 3I_3 = 15 \text{ --- (2)}$$

$$I_2 = 11A \checkmark$$

$$I_3 = 17A \checkmark$$

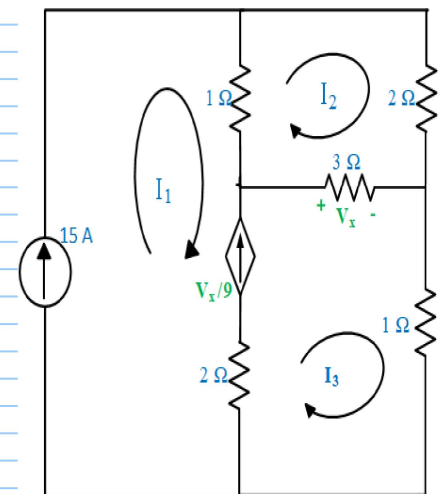


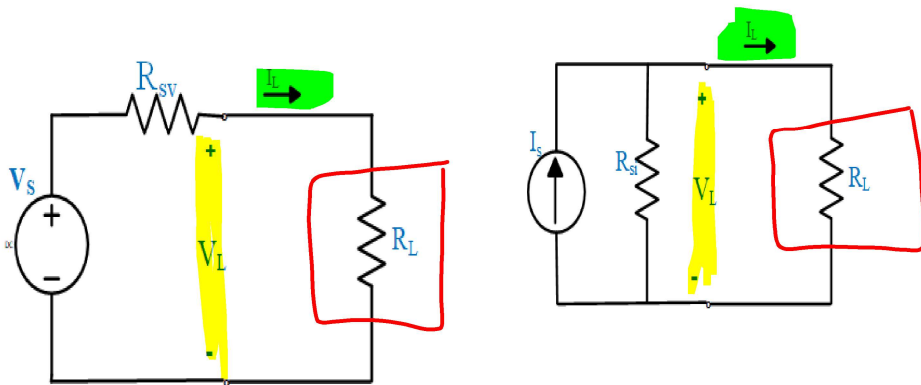
Fig. 9.16 Mesh with dependent current source.

Node or mesh: How to choose?

- Use the one with fewer equations.
- Use the method you like best.

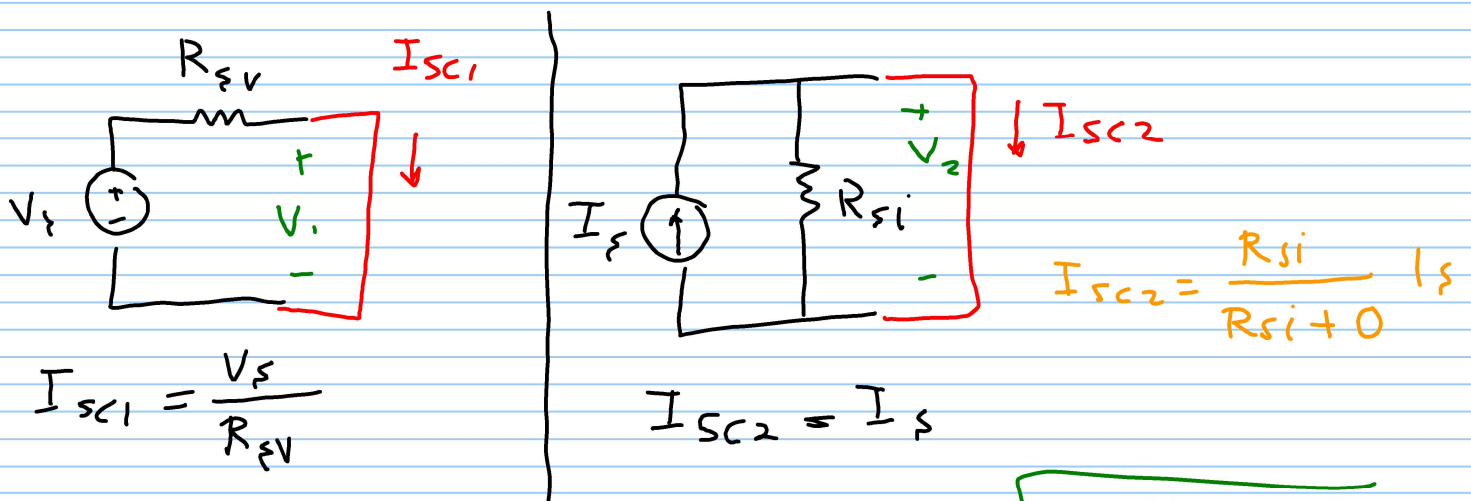
slides

Source Transformation



Two sources are equivalent, if each produces identical current and identical voltage in any load which is placed across its terminal.

↳ let $R_L = \text{Zero}$ (short circuit s.c.)



$$I_{sc1} = \frac{V_s}{R_{sv}}$$

$$I_{sc2} = I_s$$

$$I_{sc2} = \frac{R_{si}}{R_{si} + 0} I_s$$

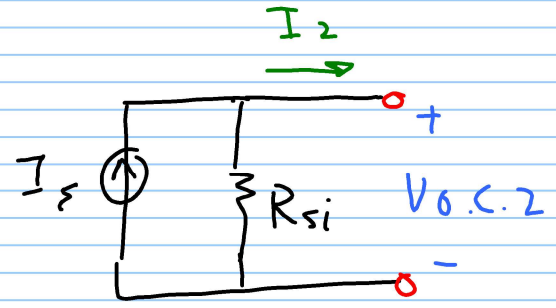
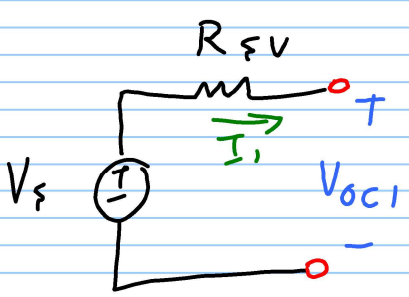
for $I_{sc1} = I_{sc2}$

$$I_s = \frac{V_s}{R_{sv}}$$

$$V_1 = V_2 = \text{Zero}$$

let $R_L = \infty$ (open circuit)

$I_1 = I_2 = \text{zero}$

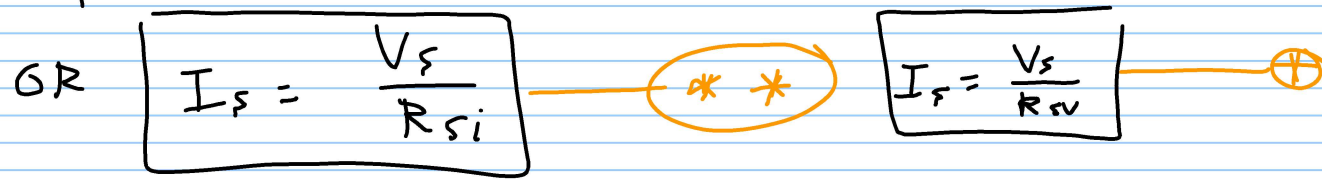


$V_{oc1} = V_s$

$V_{oc2} = I_s R_{si}$

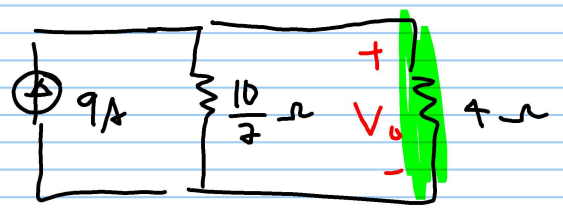
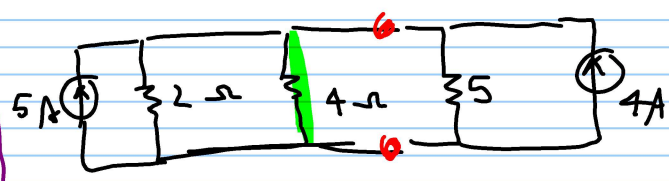
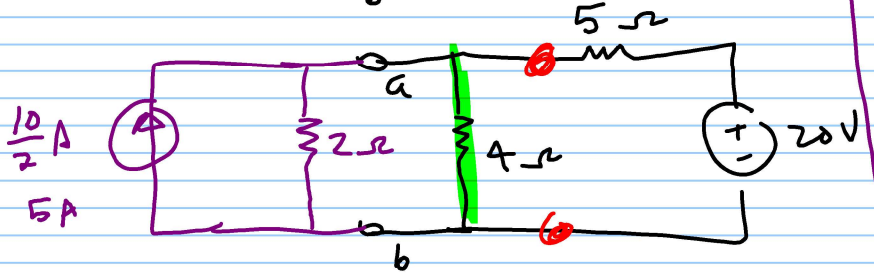
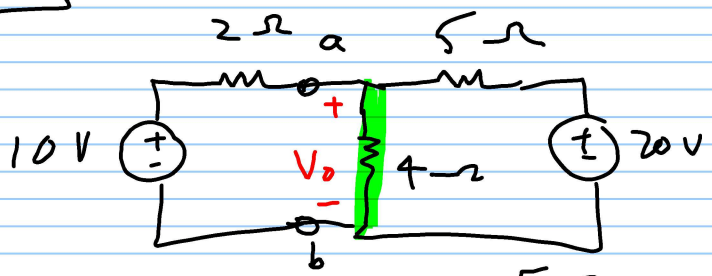
For $V_{oc1} = V_{oc2}$

$V_s = I_s R_{si}$



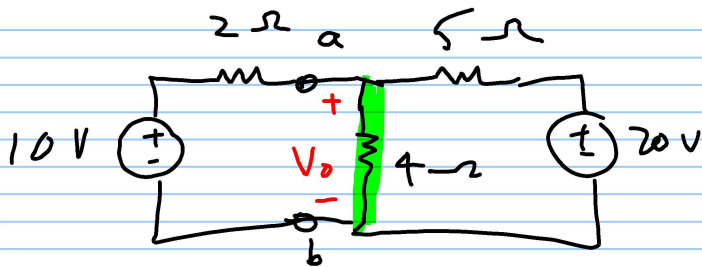
$R_{si} = R_{sV}$

Ex 1 find V_o



$\left(\frac{10/7}{10/4 + 4} \cdot 9 \right) (4) = V_o$

$V_o = 9.47 \text{ Volt}$



OR
Nodal

$$\frac{V_o - 10}{2} + \frac{V_o}{4} + \frac{V_o - 20}{5} = 0$$

$$V_o \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{5} \right) = \frac{10}{2} + \frac{20}{5}$$

$$V_o (0.95) = 9$$

$$V_o = 9.473 \text{ Volt.}$$

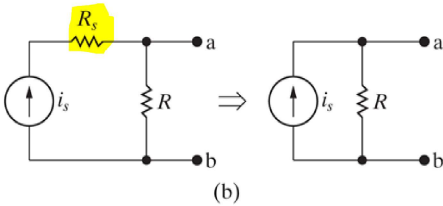
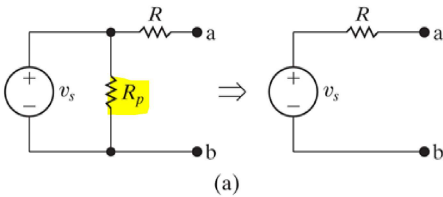


Figure: 04-39a,b

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↙
Thevenin &
Norton eq. circuit

- The two circuits depicted in Fig. 4.39(a) are equivalent with respect to terminals a,b because they produce the same voltage and current in any resistor RL inserted between nodes a,b.
- The same can be said for the circuits in Fig. 4.39(b).

Example: Find V_o using source transformation

neglect

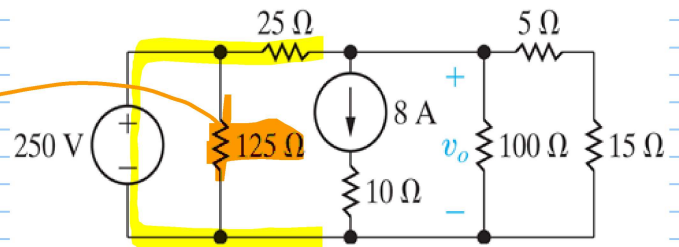
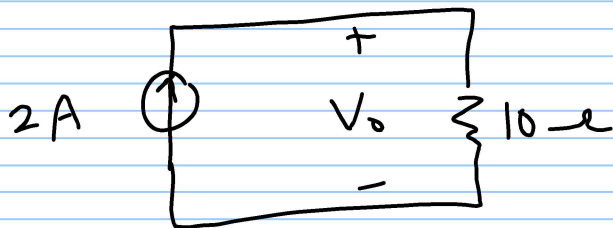
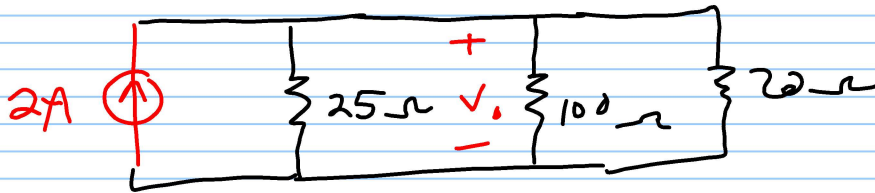
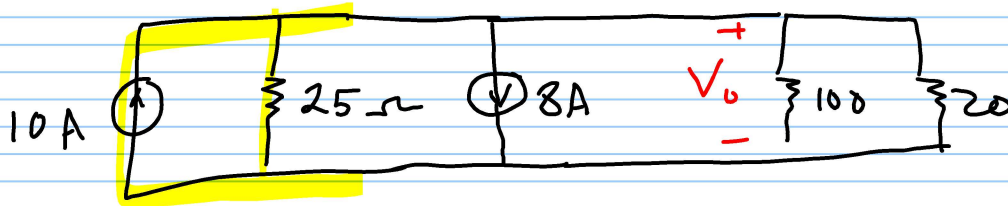
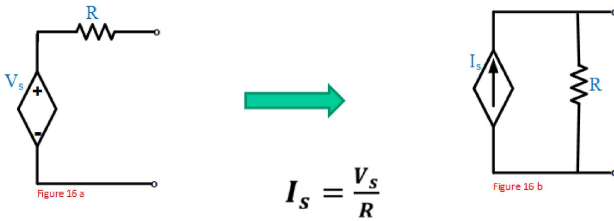


Figure: 04-40Ex4.9
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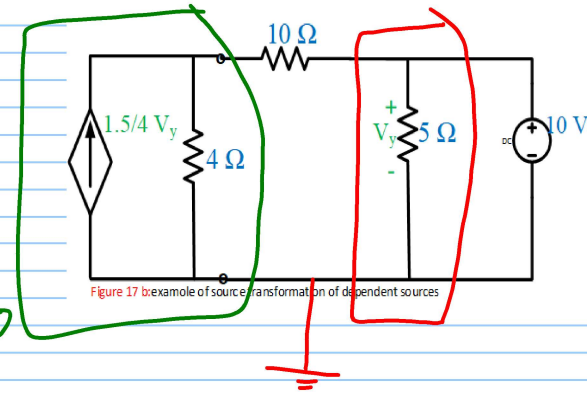
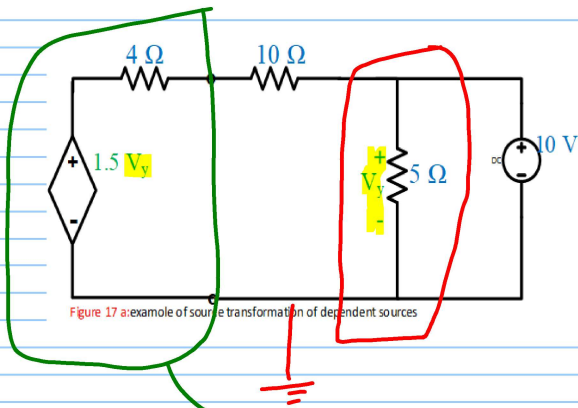


$$V_o = (2A)(10\Omega) = 20 \text{ Volt.}$$

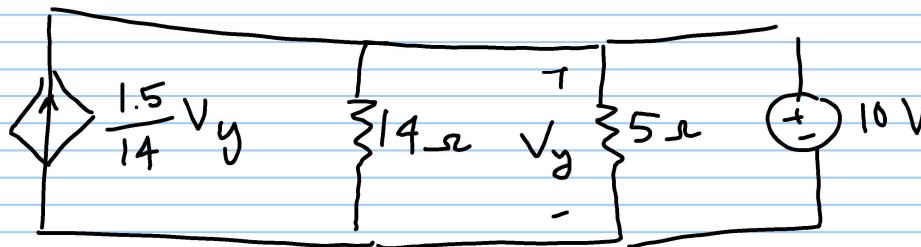
Dependent sources



The control variable must be outside the transformation.



OR



The Superposition Theorem

In a linear network, the voltage across or the current through any element may be calculated by adding algebraically all the individual voltages or currents caused by the **separate independent sources acting alone**, i.e. with:

- ② 1) All other **independent voltage** sources replaced by short circuits.
- ③ 2) All other **independent current** sources replaced by open circuits.
- ① ➤ **Dependent sources are left intact because they are controlled by circuit variables.**

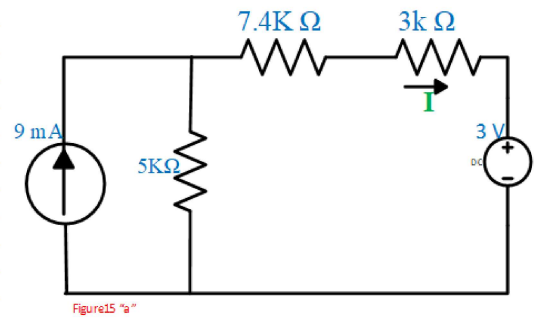


Steps to apply superposition principle

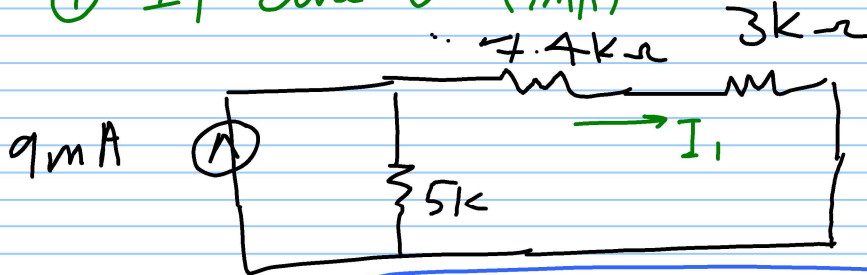
- Turn off all independent sources except one source. Find the output (voltage or current) due to that source using nodal, mesh, ...
- Repeat step 1 for each of the other independent sources.
- Find the total contribution by adding algebraically all contributions due to each independent sources.

EX)

find (I) using
superposition

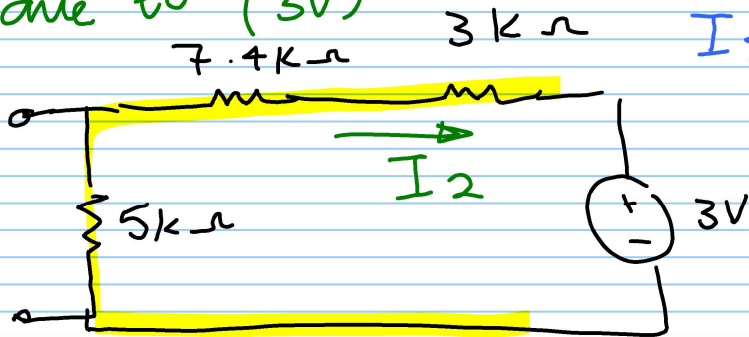


① I_1 due to (9mA)



$$I_1 = \frac{5k}{5k + 7.4k + 3k} \times 9m$$
$$= 2.922 \text{ mA}$$

② I_2 due to (3V)



$$I_2 = \frac{-3}{15.4k}$$
$$= -0.194 \text{ mA}$$

$$\therefore I = I_1 + I_2$$
$$= 2.992 - 0.194$$
$$= 2.728 \text{ mA}$$

Use superposition to solve for i_x

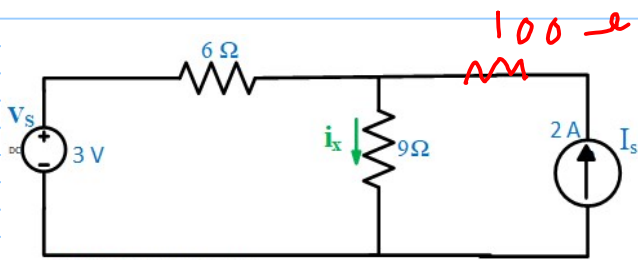
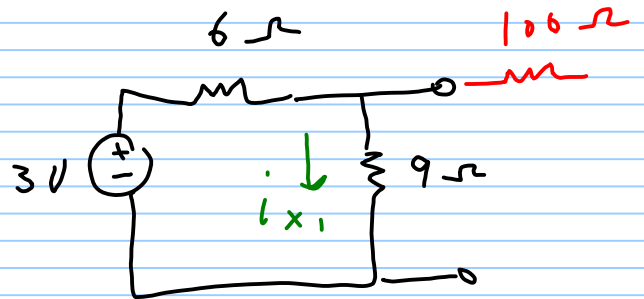


Figure 1. "a" example of superposition

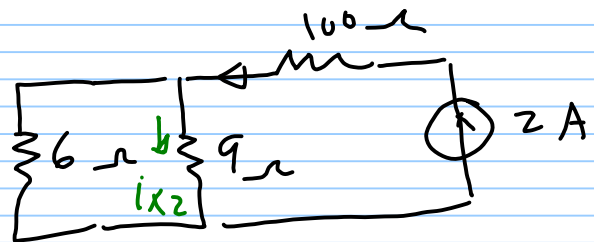
① i_{x1} due to (3V)

$$i_{x1} = \frac{3}{9+6} = 0.2 \text{ A}$$



② i_{x2} due to 2A

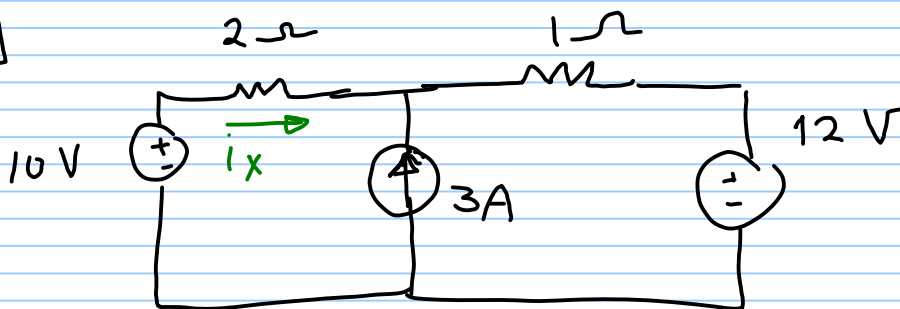
$$i_{x2} = \frac{6}{6+9} 2 \text{ A} = 0.8 \text{ A}$$



$$\text{So } i_x = i_{x1} + i_{x2} = 0.2 + 0.8 = 1 \text{ A}$$



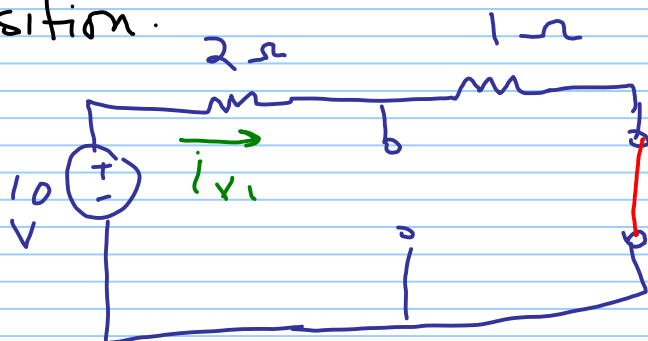
EX 1



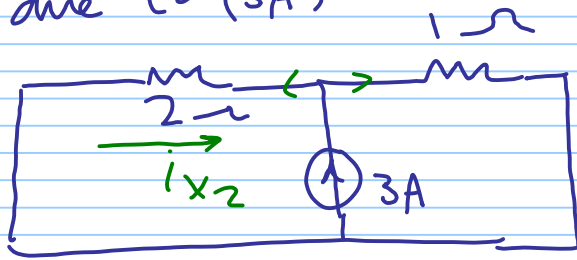
find I_x using superposition.

① i_{x1} due to (10V)

$$i_{x1} = \frac{10}{3} \text{ A}$$

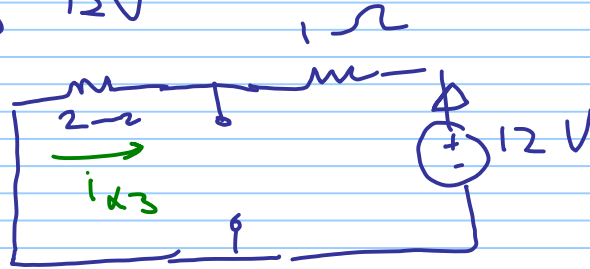


② i_{x2} due to (3A)



$$i_{x2} = \frac{1}{3} \cdot 3 = -1 \text{ A}$$

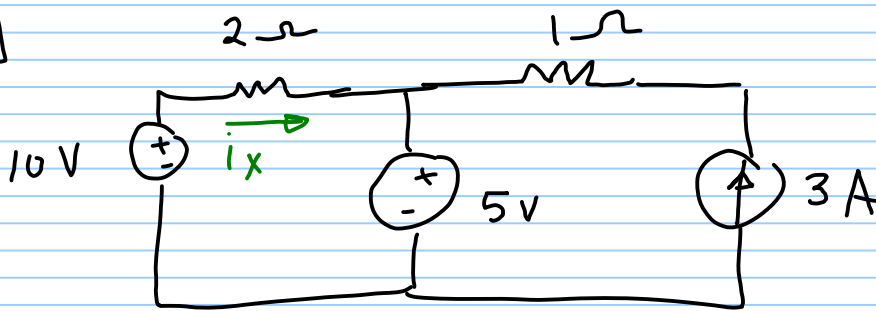
③ i_{x3} due to 12V



$$i_{x3} = -\frac{12}{3} = -4 \text{ A}$$

$$\begin{aligned} \therefore I_x &= i_{x1} + i_{x2} + i_{x3} \\ &= 3.33 - 1 - 4 \\ &= -1.666 \text{ A} \end{aligned}$$

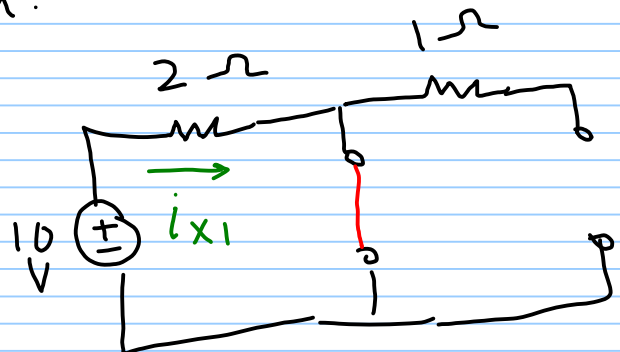
EX1



find I_x using superposition.

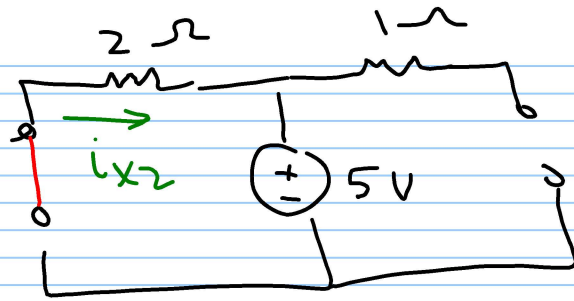
① i_{x1} due to (10V)

$$i_{x1} = \frac{10}{2} = 5 \text{ A}$$



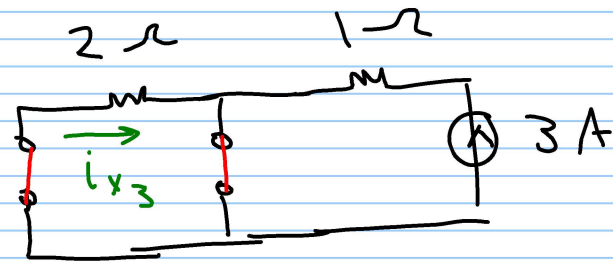
② i_{x_2} due to (5V)

$$i_{x_2} = -\frac{5}{2} = -2.5A$$



③ i_{x_3} due to (3A)

$$i_{x_3} = \text{Zero}$$



$$\begin{aligned} \text{or } i_x &= i_{x_1} + i_{x_2} + i_{x_3} \\ &= 5 - 2.5 + 0 \\ &= 2.5A \end{aligned}$$

Find i_x using superposition

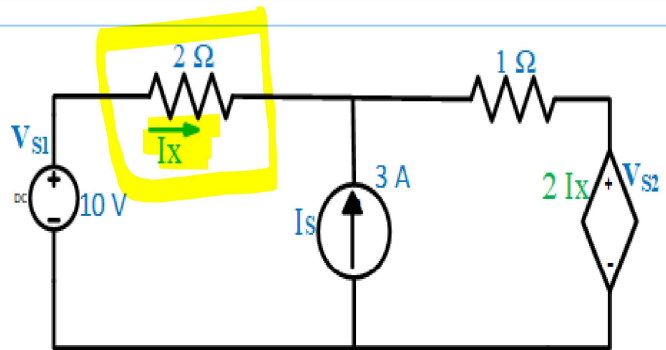


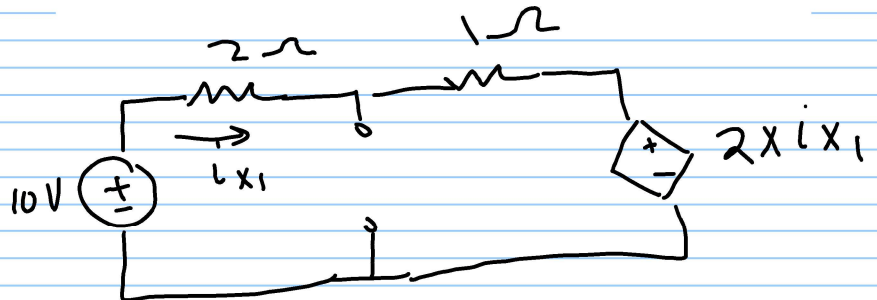
Figure 1.11: Superposition with a dependent source

① i_{x_1} due to 10V

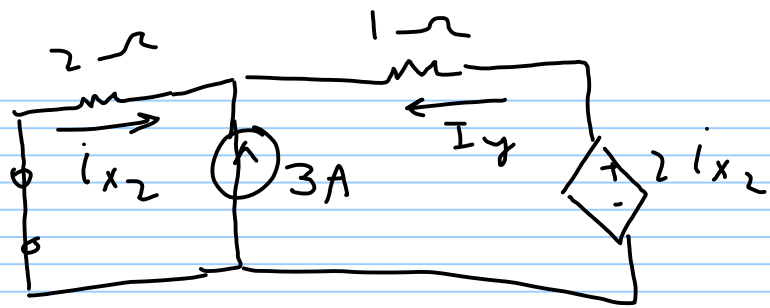
KVL

$$-10 + 3i_{x_1} + 2i_{x_1} = 0$$

$$i_{x_1} = 2A$$



② i_{x2} due to (3A)



KVL

$$2i_{x2} - I_y + 2i_{x2} = 0$$

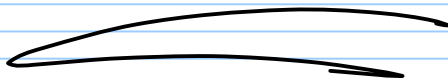
$$4i_{x2} - I_y = 0$$

KCL $I_{x2} + I_y = 3$

$$5i_{x2} = 3$$

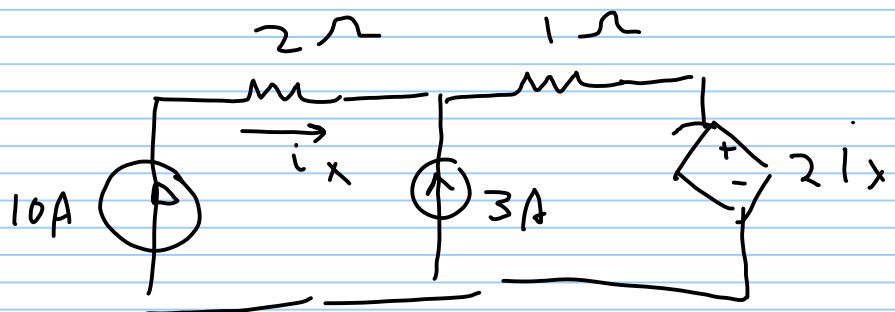
$$i_{x2} = 3/5 = -0.6 \text{ A}$$

$$\therefore I_x = i_{x1} + i_{x2} = 2 - 0.6 = 1.4 \text{ A}$$



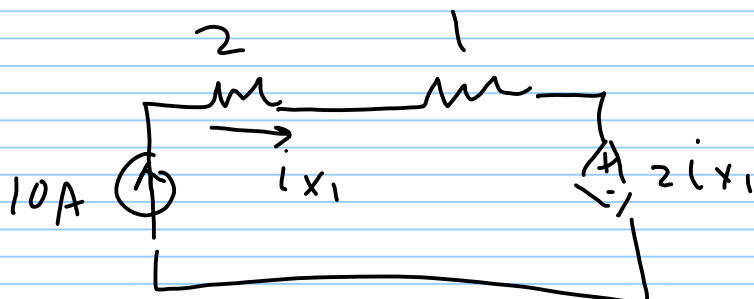
Ex Find i_x

$$i_x = 10 \text{ A}$$



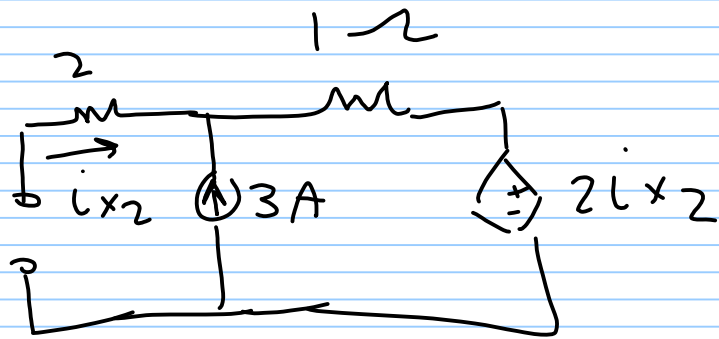
① i_{x1} due to (10A)

$$i_{x1} = 10 \text{ A}$$



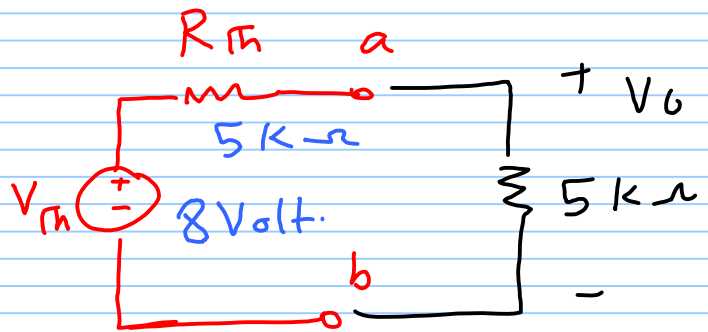
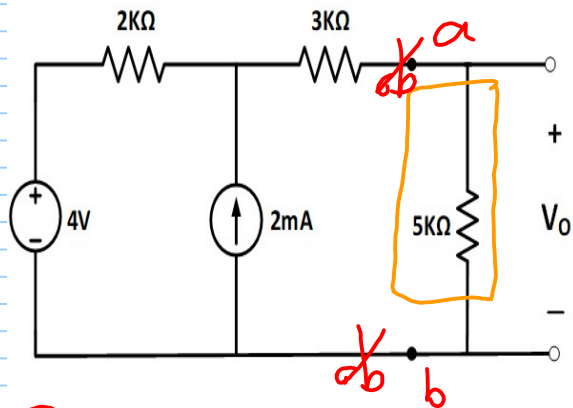
(2) i_{x2} due to (3A)

$i_{x2} = 0$



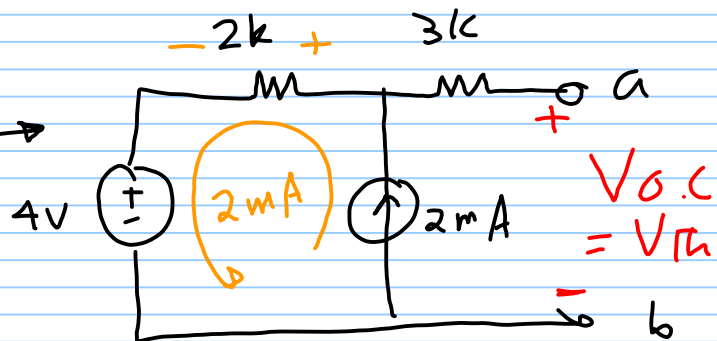
Thevenin & Norton eq. circuits

Example: Find V_o using thevenin's equivalent circuit



V_{Th}, R_{Th}

→ to find V_{Th}
 $V_{Th} = V_{O.C.}$



∴ $V_{Th} = (2k)(2m) + 4$
 $= 8 \text{ Volt.}$

How to find R_{Th} .

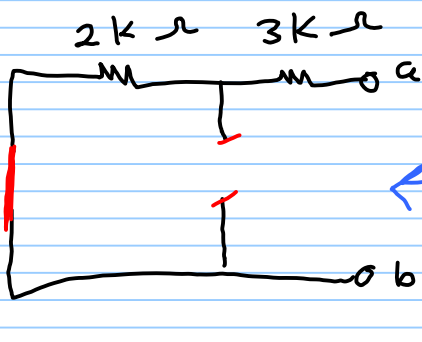
(No dependent sources)

Best method

Kill all independent sources \rightarrow set to zero

all indep. V sources $0V \rightarrow$ S.C. (short circuit)

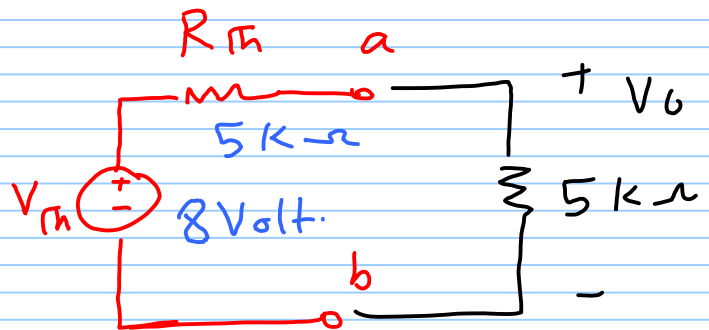
all indep. I sources $0A \rightarrow$ O.C. (open circuit)



$$R_{Th} = 3k + 2k = 5k\Omega$$

Just
- w/ r/p

$$V_o = \frac{5}{5+5} \times 8 = 4V$$



to find R_{Th} using $V_{o.c.}$ & $I_{s.c.}$ \rightarrow dep. sources \checkmark
 \rightarrow indep. sources \checkmark

(Note: you can use this method when there

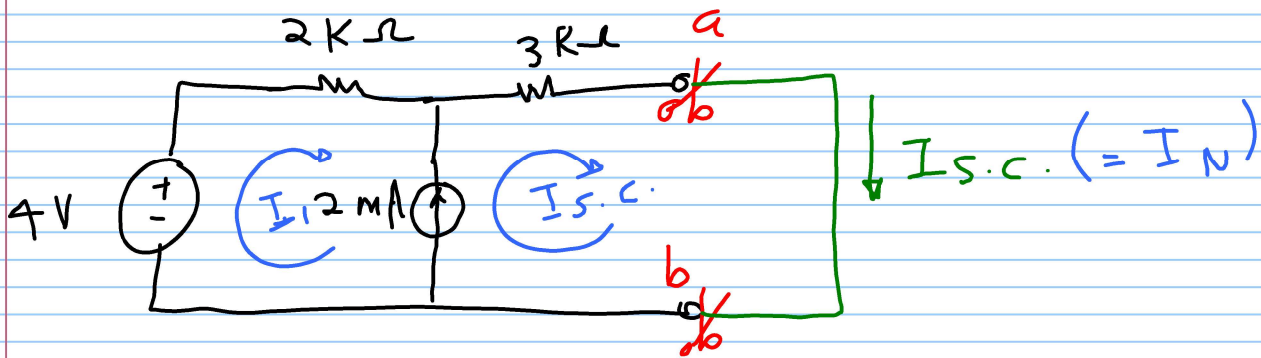
is NO dep. sources, But \rightarrow time consuming!!)

method (2)

$V_{o.c.} = 8V$ (the same as before)

$$R_{Th} = \frac{V_{o.c.}}{I_{s.c.}}$$

Find $I_{s.c.}$



$$2k \times (I_{s.c.} - I_1 = 2mA) \quad \text{--- (1)}$$

$$-4 + 2k I_1 + 3k I_{s.c.} = 0$$

$$(3k I_{s.c.} + 2k I_1 = 4) \quad \text{--- (2)}$$

$$5k I_{s.c.} = 8$$

$$I_{s.c.} = \frac{8}{5} \text{ mA}$$

$$= I_N$$

$$\begin{aligned} \therefore R_{Th} &= \frac{V_{o.c.}}{I_{s.c.}} = \frac{8}{8/5 \text{ m}} \\ &= 5k\Omega \end{aligned}$$

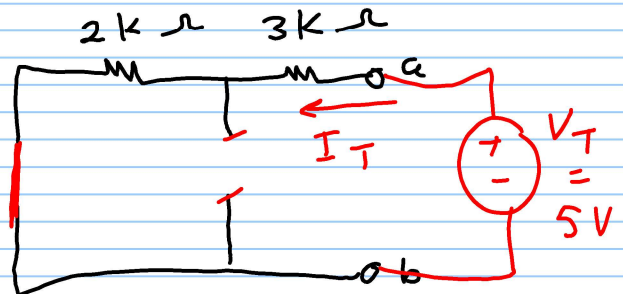
method (3) R_{Th} using test sources

$$R_{Th} = \frac{V_T}{I_T} \quad \left| \begin{array}{l} \text{all indep. sources are} \\ \text{set to zero} \end{array} \right. \quad \begin{array}{l} \checkmark \\ \text{dep. \& indep} \\ \text{sources} \\ \checkmark \end{array}$$

$$\text{let } V_T = 5V$$

$$\text{then } I_T = \frac{V_T}{R_{eq}} = \frac{5}{5k\Omega} = 1mA$$

$$\therefore R_{Th} = \frac{5}{1m} = 5k\Omega \quad \checkmark$$



* methods to Find R_{Th} :-

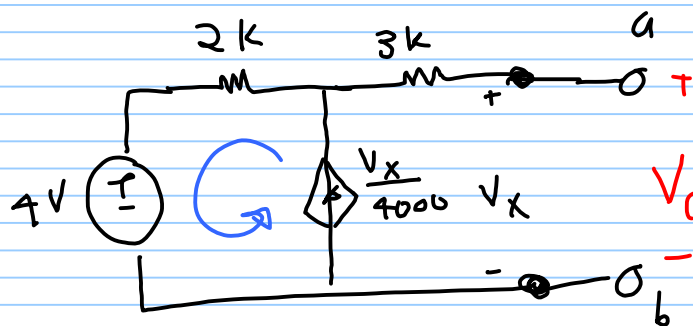
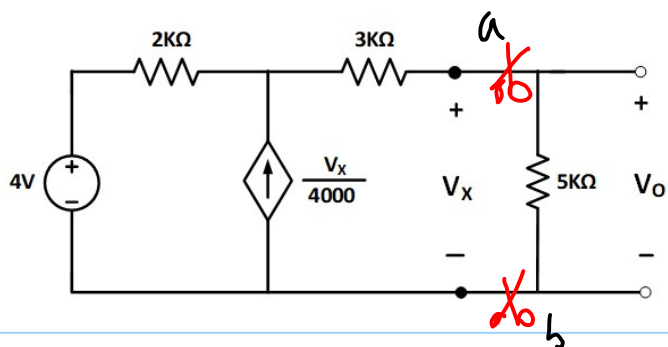
method 1 | if there is no dep. sources (best method) (easy one)
 → kill all indep. sources, then find R_{Th} (series/parallel combination)

method 2 | $R_{Th} = \frac{V_{O.C.}}{I_{S.C.}}$ General method

can be applied if the circuit has dep. sources (also it can be applied if the circuit does not have dep. sources)

method 3 | $R_{Th} = \frac{V_T}{I_T}$ | all indep. sources = 0 (dep. sources are left intact.)

Find V_o using Thevenin's theorem



$V_{Th} = V_{o.c.}$

for this circuit

$V_{Th} = V_{o.c.} = V_x !$

$$V_{o.c.} = \left(\frac{V_x}{4000} \right) 2k + 4V = \left(\frac{V_{o.c.}}{4000} \right) 2k + 4$$

$$V_{o.c.} = \frac{1}{2} V_{o.c.} + 4$$

$V_{o.c.} = 8 \text{ Volt.}$

to find R_{Th} , method 1 X

method 2 or method 3 ✓ (dep. source)

method 2 | $V_{o.c.} = 8V$.

find $I_{s.c.}$.

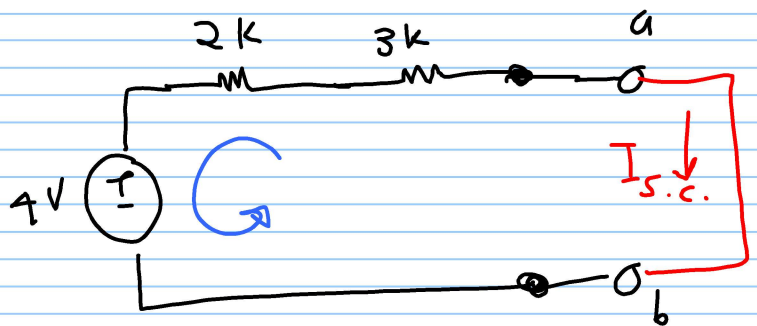
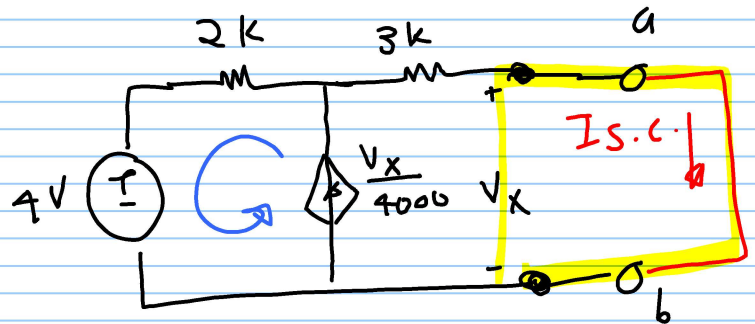
$V_x = \text{Zero!!}$

$\frac{V_x}{4000} = 0A (o.c.)$

∴ $I_{s.c.} = \frac{4}{5k} = 0.8mA$

∴ $R_{Th} = \frac{V_{o.c.}}{I_{s.c.}} = \frac{8}{0.8m}$

$= 10k\Omega$



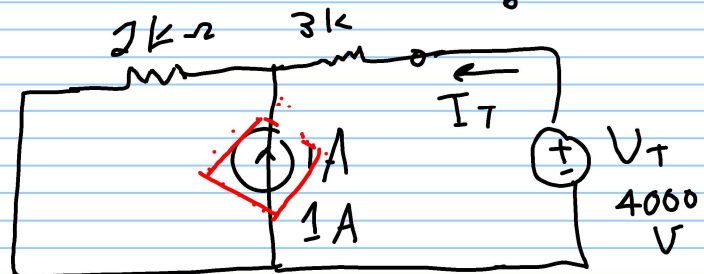
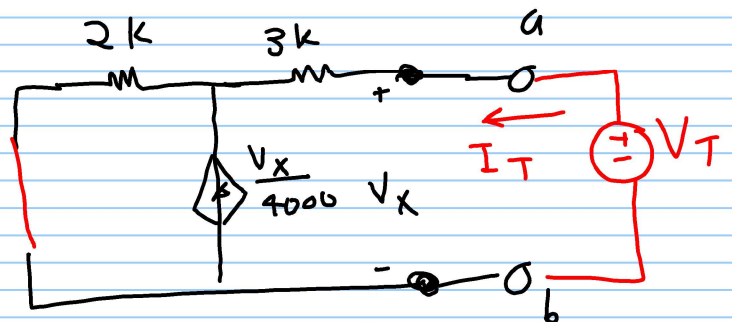
method 3 | $R_{Th} = \frac{V_T}{I_T}$ | all indep. sources killed

let $V_T = 4000V$

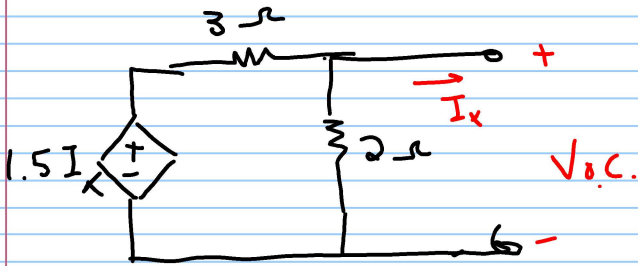
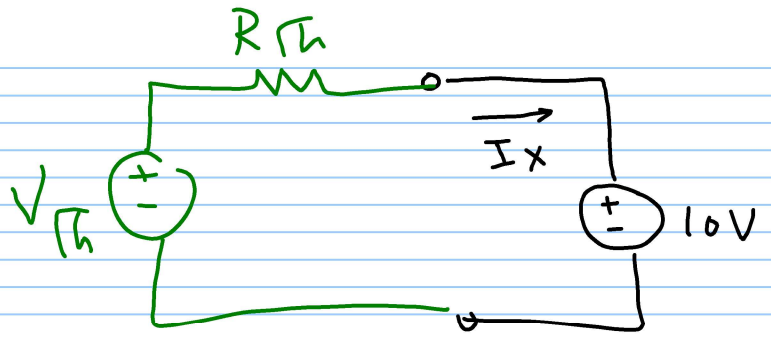
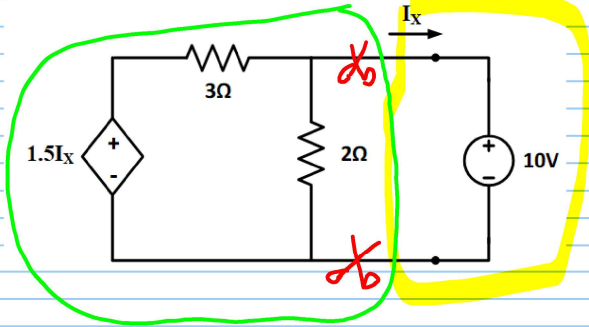
∴ $I_T = \frac{4000 - 2000}{5000}$

$= 0.4A$

∴ $R = \frac{4000}{0.4} = 10k\Omega$



Find I_x using Thevenin's theorem

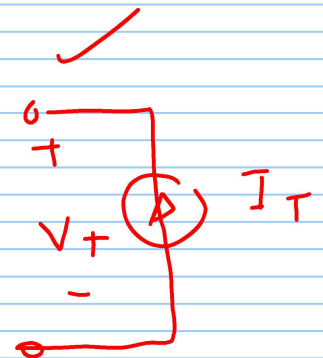
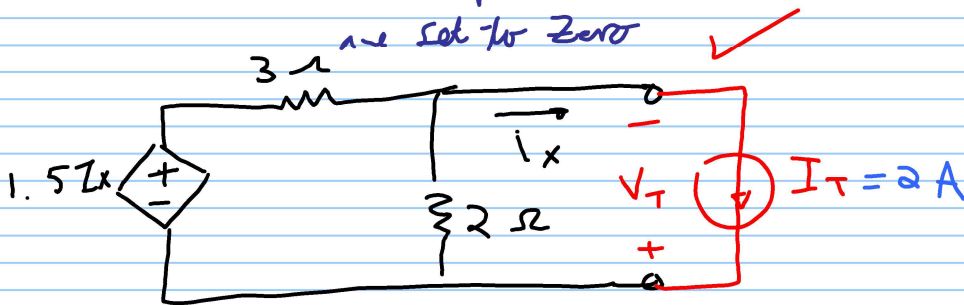


Since there is only dep. sources
 $\therefore V_{O.C.} = V_{Th} = 0$

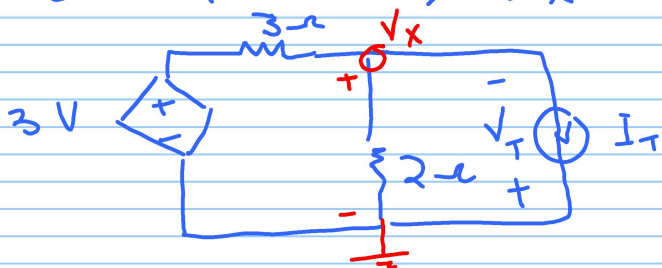
method (2) X

✓ to find R_{Th}

$$R_{Th} = \frac{V_T}{I_T} \quad \text{all indep. sources are set to zero}$$



let $I_T = 2A$, $I_x = I_T$



Nodal

$$\frac{V_x - 3}{3} + \frac{V_x}{2} + 2 = 0$$

$$V_x \left(\frac{1}{3} + \frac{1}{2} \right) = -2 + 1$$

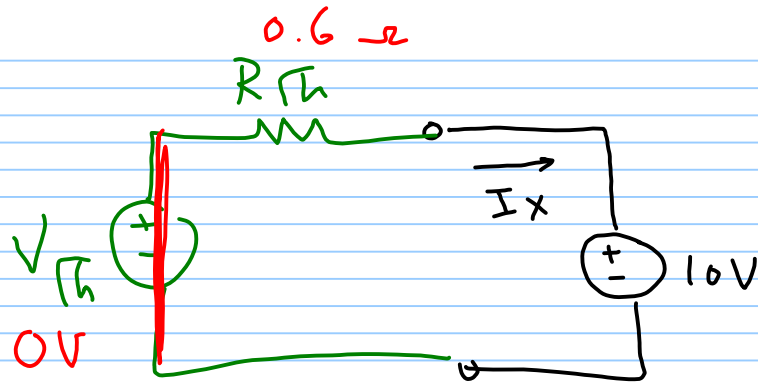
$$V_x = -\frac{6}{5} V$$

$$\therefore V_T = -V_x = \frac{6}{5} V$$

$$\therefore R_{Th} = \frac{V_T}{I_T} = \frac{6/5}{2} = 0.6 \Omega \quad \checkmark$$

$$I_x = \frac{-10}{0.6}$$

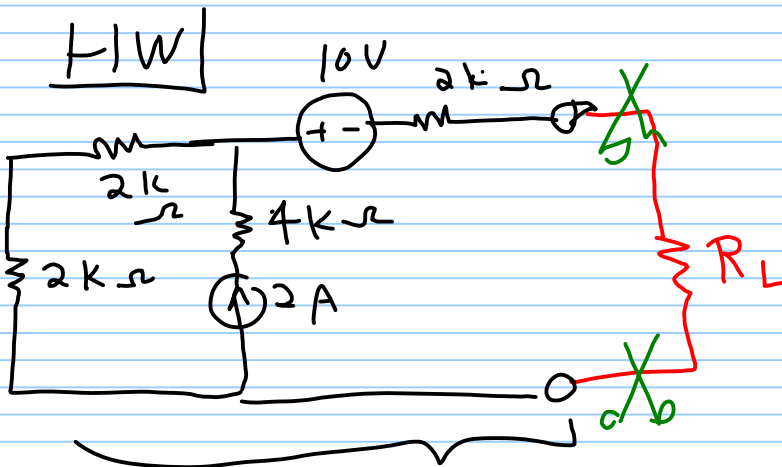
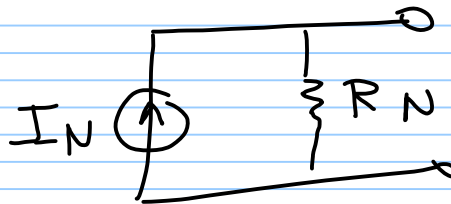
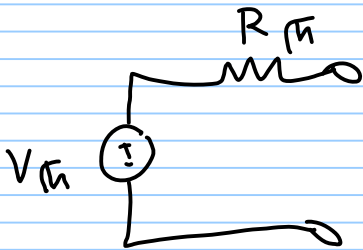
$$= -16.67 \text{ A}$$



$$R_{Th} = R_N$$

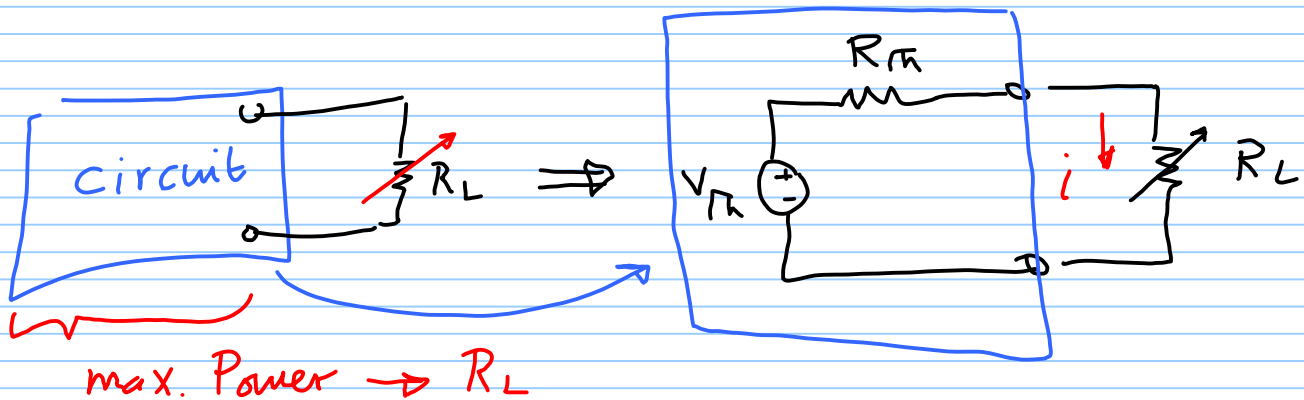
$$V_{Th} \rightarrow V_{o.c.}$$

$$I_N \rightarrow I_{s.c.}$$



find the thevenin of circuit.

4.12 Maximum Power Transfer



$$P = i^2 R_L = \left(\frac{V_{th}}{R_L + R_{th}} \right)^2 R_L$$

P is max when

$$\frac{dP}{dR_L} = \text{Zero}$$

$$P = \frac{V_{th}^2 R_L}{(R_L + R_{th})^2}$$

$$\frac{dP}{dR_L} = V_{th}^2 \left[\frac{(R_{th} + R_L)^2 - R_L \times 2 \times (R_L + R_{th})}{(R_{th} + R_L)^4} \right] = 0$$

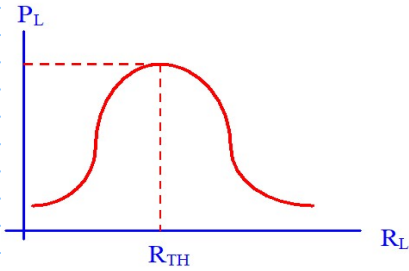
$$(R_{th} + R_L)^2 = 2 R_L (R_L + R_{th})$$

$$R_{th}^2 + R_L^2 + 2R_L R_{th} = 2R_L^2 + 2R_L R_{th}$$

$$R_L = R_{th}$$

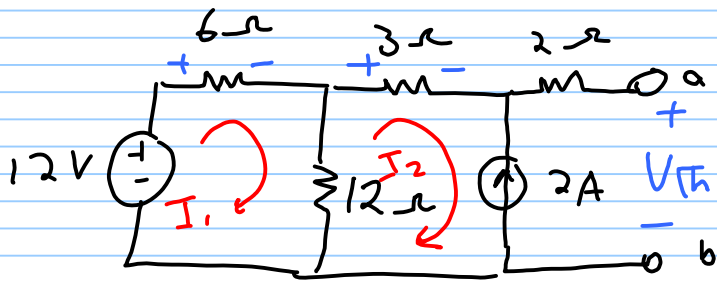
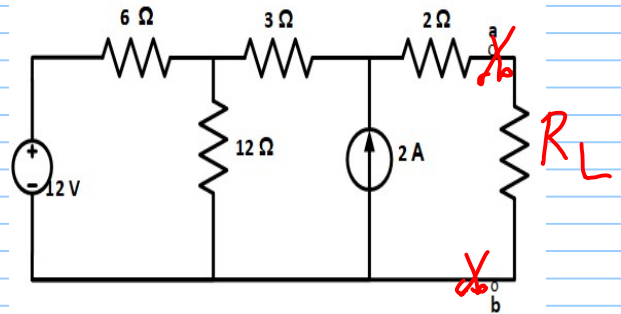
\rightarrow The max. power transfer occurs when the load resistance $R_L = R_{th}$ or $P_{max} = \left(\frac{V_{th}}{2R_L} \right)^2 R_L$

$$P_{\max} = \frac{V_{\text{th}}^2}{4 R_L}$$



Find the value of R_L for maximum power transfer in the circuit shown .

Find the maximum power .



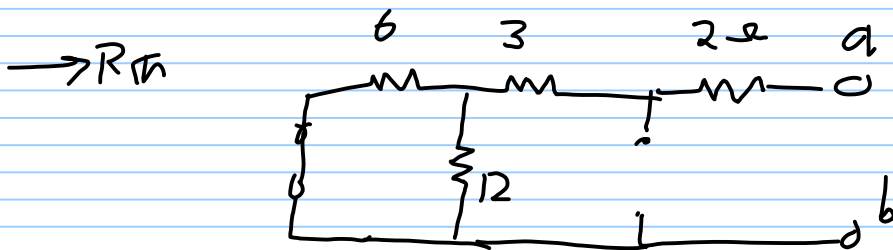
$$I_2 = -2A$$

$$-12 + 6I_1 + 12(I_1 - I_2) = 0$$

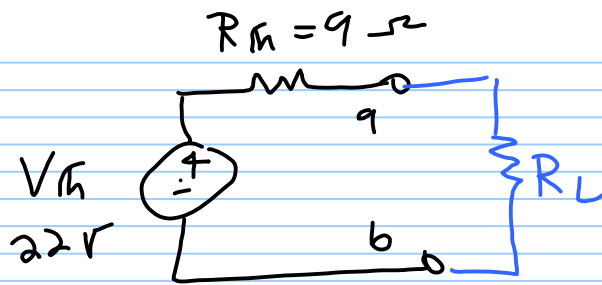
$$-12 + 6I_1 + 12I_1 + 12 \times 2 = 0$$

$$I_1 = -\frac{2}{3}A$$

$$V_{\text{th}} = -3 \times I_2 - 6 \times I_1 + 12 = \underline{\underline{22V}}$$



$$R_{\text{th}} = (6 // 12) + 3 + 2 = 9\Omega$$



∴ for max. Power transfer

$$R_L = R_{th} = 9 \Omega$$

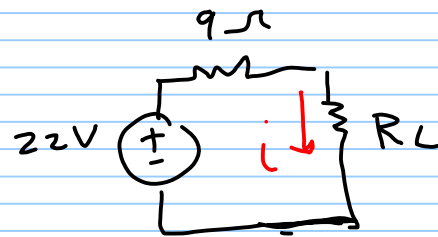
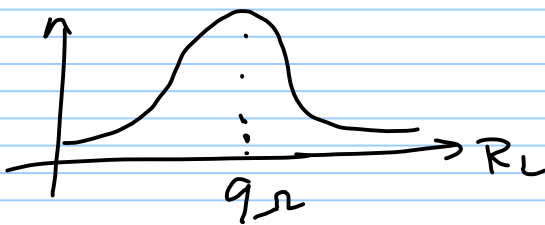
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = 13.44 \text{ W}$$

if $R_L = 6 \Omega$

$$P_{R_L} = i^2 R_L$$

$$= \left(\frac{22}{9+6} \right)^2 \times 6$$

$$P = 12.9 \text{ W}$$



if $R_L = 13 \Omega$

$$P_{R_L} = \left(\frac{22}{9+13} \right)^2 \times 13$$

$$= 13 \text{ W}$$